

# 4 Meaning and Logic

## 4.1 Logical basics

### ■ Law of Contradiction

The same attribute cannot at the same time both belong and not belong to the same subject in the same respect.

Aristotle, *Metaphysics*, 1005b, p. 262.

- I.e. a sentence, in a certain reading, cannot be true and false at the same time.

### ■ The Principle of Polarity

In a given CoU, with a given reading, a declarative sentence is either true or false. *Aristotle*

- The PoP entails the PoC, i.e. if a declarative in a given reading is either true or false, then the attribute in it cannot at the same time both belong and not belong to the subject in the same respect.

### ■ Law of the Excluded Middle (*Tertium non datur*)

There is no third possibility. There is only truth or falsity, and no others.

## ■ Truth value

To subsume the properties be true or false, one speaks of the *truth value* of a sentence. A sentence has the truth value 'true' if it is true and it has the truth value 'false' if it false.

- The truth value of a sentence depends on the agreement between the descriptive meaning of the sentence, the truth conditions, and the facts given by the given CoU. If the CoU fulfills the sentence meaning, i.e. there is fact corresponding to it, the sentence is true. If not, the sentence is false.

## ■ Assertion

The ***utterance of a declarative sentence*** is understood as tacitly denying that the contrary is true (cf. the Principle of Polarity). Thus, any statement that one can express in a language is polarized.

## ■ Negation

Negation reverses the truth value of a sentence, it makes a true sentence false and a false sentence true.

(1) John knows the solution.

It is not true that John knows the solution.

John does not know the solution.

(2) John does not know the solution

It is not true that John does not know the solution

John knows the solution

John does know the solution

## ■ Negation of a Sentence

If  $\sigma$  is a sentence that is not negated itself, then its negation is true whenever  $\sigma$  is false and false whenever  $\sigma$  is true.

## ■ Negation forms

- adding the auxiliary *do* to the verb phrase and *not* to *do*

(3) Mary reads a book

Mary does not read a book

- adding *not* to an auxiliary verb

(4) Mary is reading a book

Mary is not reading a book.

- adding *not* to a quantifier expressions

(5) M is always here.

M is not always here

(6) Everybody likes that.

Not everybody likes that.

- o substituting a positive expression by its negative counterpart  
(Negative polarity items)

(7) Mary is already there.

Mary isn't there yet

(8) Mary is sometimes there.

Mary is never there.

(8) Mary is still asleep.

Mary is no more asleep.

(9) Mary owns some books.

Mary does not own any books.

## 4.2 Logical properties of sentences

### ■ Logical contingent sentences

A sentence  $\sigma$  is *contingent* if it is neither necessarily true or necessarily false.

Or,  $\sigma$  is contingent if there is a CoU where it is true and if there is a CoU where it is false.

- There are two kinds of sentences which are not contingent
  - ***logically true sentences***
    - (*tautological sentences*)
  - ***logically false sentences***
    - (*contradictory sentences*)

## ■ Tautologies

A sentence  $\sigma$  is logically true if it is true in all possible CoUs.

(10) Either DD is a duck or he is not a duck.

The clause is true due to the Principle of Polarity.

(11) Every duck is a duck.

The clause is true due to the structure of the sentence

(12) Ducks are birds.

The clause is true due to the meanings of the words *duck* and *bird*.

(13) Two times seven equals fourteen.

The clause is true due to mathematical rules

## ■ Logical contradictions

A sentence  $\sigma$  is logically false (inconsistent, contradictory) if it is false in all possible CoUs.

(14) DD is a duck and DD is not a duck.

The clause is false due to the Law of Contradiction.

(15) DD is neither a duck nor is he not a duck.

The clause is false due to the Law of the Excluded Middle

(16) Ducks are plants.

The clause violates the semantic rules of English.

(17) Two times seven is twenty-seven.

The clause violates the rules of mathematics.



## ■ Logical consistent sentences

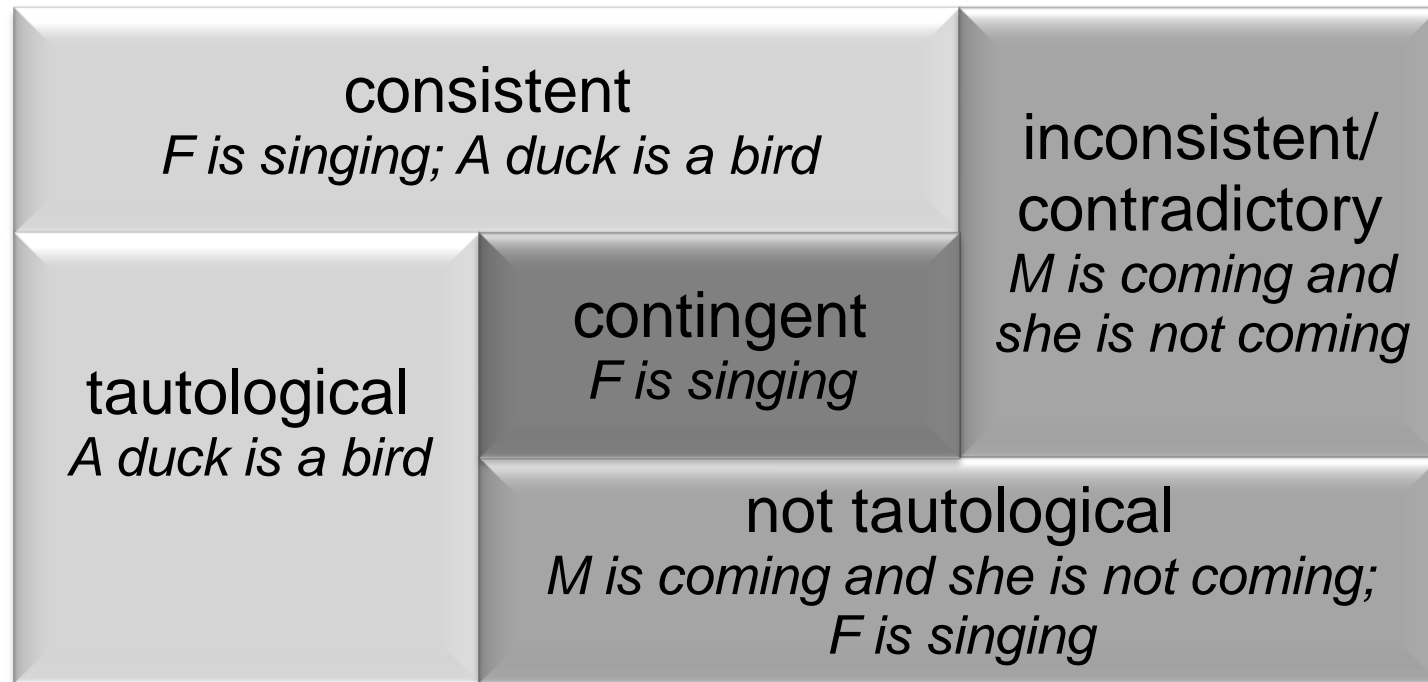
A sentence  $\sigma$  is *consistent* if there is a CoU where it is true.

- Contingent and logically true (tautological) statements are consistent.

(18) Every duck is a duck.

(19) DD is a duck.

- semantic properties of statements



## 4.3 Logical relations between sentences

- entailment (implicativity)
- Logical equivalence
- Logical contrariety (exclusion, incompatibility, inconsistency)
- Logical contradiction (anti-implicativity)

## ■ Logical entailment (implication)

- A logically entails B / B logically follows from A / A logically implies B /  $A \rightarrow B$ , iff necessarily, if A is true, B is true.

<b>A</b>	<b>B</b>	<b>A <math>\rightarrow</math> B</b>	
1	1	1	
1	0	0	by logical entailment
0	1	1	
0	0	1	

- If DD is a duck, he is a bird.
- If it is raining heavily, it is raining.
- If Ann is a sister of my mother, Ann is an aunt of mine.
- If Frank knows that Maria is sleeping, she is sleeping.
- If it rains heavily, the streets are wet.

- **Entailment reversing**

$A \rightarrow B$

$\neg B \rightarrow \neg A$       ***Contraposition***

$(A \rightarrow B) \leftrightarrow (\neg B \rightarrow \neg A)$

If A entails B, then A is necessarily false, if B is false.

If DD is not a bird, he cannot be a duck.

If it is not raining, it cannot be raining heavily

If the streets are not wet, it has not rained heavily

- **No logical entailment**

If Mary is John's mother, she is the wife of his father.

If John said he is tired, he is tired.

If the beer is in the fridge, the beer is cool.

## ■ Logical equivalence

- A and B are logically equivalent /  $A \leftrightarrow B$   
iff A and B have equal truth values

A	B	$A \leftrightarrow B$	
1	1	1	
1	0	0	by logical entailment
0	1	0	by logical entailment
0	0	1	

- If he is the father of my mother, he is my maternal grandfather.
- If today is Monday, yesterday was Sunday.
- If the bottle is half empty, it is half full.
- If everyone will lose, nobody will win.

## ▪ Logical contrariety

### (exclusion, incompatibility, inconsistency)

- A is logically contrary to B / A logically excludes B / B is incompatible with B iff necessarily, if A is true, B is false.

A	B	A is contrary to B
1	1	0
0	1	1
1	0	1
0	0	1

It is hot excludes that it is cold.

Today is Monday excludes that tomorrow is Wednesday.

Ann is younger than Mary is incompatible with A is older than M.



## ■ logical contradiction (complementary)

- A and B are logical contradictories iff necessarily, A and B have opposite truth values.

A	B	A and B are contradictory
1	1	0
1	0	1
0	1	1
0	0	0

It is late contradicts it is not late.

Today is Saturday or Sunday is contradictory with today is Monday, Tuesday, Wednesday, Thursday or Friday.

Everyone will win contradicts someone will lose.

## 4.3 Logical relations between words

- For establishing logical relations between two expressions, we insert them into an appropriate test sentence and check the resulting logical relations

▪ count noun	<i>car</i>	x is a car
▪ mass noun	<i>mud</i>	x is mud
▪ adjective	<i>dirty</i>	x is dirt
▪ intransitive verb	<i>smell</i>	x smells
▪ transitive verb	<i>sell</i>	x sells y

- **Logical subordination** (entailment, implication)

x is a duck →

x is a bird

x enlarges y →

x changes y

x mutters  $\sigma$  →

x utters  $\sigma$

x is green →

x is colored

- **Logical equivalence** (synonymy)

x is a female adult                       $\leftrightarrow$   
x is a woman

x costs a lot                                 $\leftrightarrow$   
x is expensive

x is an unmarried young man         $\leftrightarrow$   
x is a bachelor

- **Logical contrariety** (incompatibility)

- Two terms A and B are logically incompatible iff their denotations have no elements in common.

x is a swan is incompatible with  
x is pigeon.

x doubts that  $\sigma$  is incompatible with  
x knows that  $\sigma$ .

x is cold is contrary to  
x is hot.

- **Logical complementary** (contradiction)

- Two terms A and B are logically complementary iff their denotations have no elements in common and together exhaust the set of possible cases.

x is a member contradicts

x is a non-member.

x is true contradicts

x is false.

## 4.4 Logic and Meaning

- Logical relations are not to be confused with meaning relations such as homonymy, synonymy and hyponymy
- Logical equivalence does not mean identity of meaning
- All logical notions are based on truth conditions and denotations. They only concern descriptive meaning.
- Descriptive meaning is not exhausted by logical notions.

- The semantic status of logical equivalence
  - The descriptive meaning of a sentence provides its truth conditions and determines its denotation.
    - Ich werde Sie verhaften.
    - I will arrest you.
  
    - John didn't take his car away.
    - John didn't take his fucking car away.



- Logical equivalence does not mean equal descriptive meaning,
  - logical true sentences are logical equivalent despite differences in descriptive meaning
- a. Either DD is a duck or he is not a duck.
- b. Every duck is a duck.
- c. Ducks are birds.
- d. Two times seven equals fourteen.

$a \leftrightarrow b$

$b \leftrightarrow c$

$c \leftrightarrow d$

- a. If he is the father of my mother, he is my maternal grandfather.
  - b. If today is Monday, yesterday was Sunday.
  - c. If the bottle is half empty, it is half full.
  - d. If everyone will lose, no-one will win.
- 
- a'. If he is the father of my mother, he is my maternal grandfather.

A

B

- $A \leftrightarrow B$
- $a \leftrightarrow b$
- $b \leftrightarrow c$
- $c \leftrightarrow d$

- Logical false sentences are logical equivalent despite differences in descriptive meaning

- a. DD is a duck and DD is not a duck.
- b. DD is neither a duck nor is he not a duck.
- c. Ducks are plants.
- d. Two times seven is twenty-seven.

$a \leftrightarrow b$

$b \leftrightarrow c$

$c \leftrightarrow d$

- Words with different descriptive meanings can result in logical equivalent sentences
  - a. If it is a 'großer Zeh' (big toe), it is a 'großer Onkel'.
  - b. If it is a 'Füllfederhalter', it is a 'fountain pen'.
  - c. If it is a 'bra', it is a 'kalabus bilong susu' (prison of the breasts). Tok Pisin New Guinea creole.
- Logical equivalence is not a sufficient criterion for having the same meaning.