GRANULARITY AND APPROXIMATING NUMBER PAIRS*

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1 Introduction

Natural languages have numerous means for expressing approximate quantity and measure. This includes in particular a large variety of precision regulators such as those in (1)-(4), words and morphemes that serve to specify the level of precision or imprecision at which a numerical expression is interpreted. There is a small but growing body of literature on the semantics and pragmatics of such expressions. In addition to these, Krifka (2009) demonstrates that even unmodified round numerical expressions such as *forty-five minutes* in (5) allow approximate interpretations.

(1)	What John cooked was exactly/approximately fifty tapas.	(Sauerland & Stateva 2007)
(2)	John arrived at be-gadol ('basically') / more or less 3.	(Greenberg & Ronen 2013)
(3)	Let's meet at Starbuck's at 3-ish.	(Bochnak & Csipak 2014)
(4)	Some twenty people attended the party.	(Anderson 2014)
(5)	Mary waited for forty-five minutes.	(Krifka 2009)

In this paper, I investigate another sort of approximative construction, namely **approximating number pairs** (ANPs), whose characteristics have been well described (Sigurd 1988; Pollmann & Jansen 1996; Jansen & Pollmann 2001; Langendoen 2006; Sauerland & Stateva 2007; Eriksson et al. 2010), but which have yet to receive a formal semantic analysis. Examples are the following:

(6) a. There were forty or fifty people at the party.
b. John bought fifteen or twenty books.

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What is curious about examples such as these is that they have the form of disjunctions, but their meaning is not disjunctive; rather, they express approximate ranges. That is, (6a) does not mean that there were either 40 people or 50 people at the party in question, but rather that the number attending was between roughly 40 and roughly 50 (and similarly for (6b)).

The phenomenon of ANPs is not restricted to English: similar constructions are found in languages including Dutch, German, Swedish, French and Spanish, with the two values in some cases connected via a disjunction or preposition, in other cases simply juxtaposed (a variant which is also at least marginally possible in English).

(7) Er ist für zwei drei Tage weg. GERMAN (Sauerland & Stateva 2007)
‘He is away for two or three days’

(8) a. vier, vif nationaliteiten DUTCH (Pollmann & Jansen 1996)
‘four or five nationalities’
b. tien á twaalf docenten
‘ten or twelve teachers’

(9) Había diez o veinte personas en clase esta mañana. SPANISH
‘There were ten or twenty people in class this morning’

I will propose a semantic analysis of ANPs based on the notion of scale granularity (Krifka 2007, 2009), here is given a novel implementation in terms of sets of alternatives to a measure expression. Arguing from facts relating to this construction as well as other data, I will further make a case for a granularity-based theory of numerical (im)precision more generally

The structure of the paper is as follows. In Section 2, I present the crucial facts about ANPs in comparison to other range-denoting expressions, drawing on data reported in the literature as well as a newly conducted experiment and corpus analysis. In Section 3, I review two leading recent theories of imprecision, with a view to assessing how they fare in accounting for the previous data. Section 4 presents a novel granularity-based theory of numerical imprecision, which takes its inspiration from the metaphor of the ruler as a measuring instrument. Section 5 then applies this theory to ANPs and related constructions, and Section 6 concludes by considering some potential areas for extension.

2 Characteristics of Approximating Number Pairs

Approximating number pairs are superficially similar to other range-based approximative expressions, as evidenced by the resemblance in meaning of the examples in (10):

(10) a. There were <u>40 or 50</u> people at the public meeting.
b. There were <u>between 40 and 50</u> people at the public meeting.
c. There were <u>40 to 50</u> people at the public meeting.

A closer examination, however, reveals some important differences.

First, only certain pairs of numbers produce well-formed ANPs. The examples in (11) are all felicitous on an approximation reading. Those in (12), by contrast, are either outright infelicitous or allow only a 'true disjunction' reading.

(11) a. There were 5 or 6 people at the public meeting.
b. …10 or 12…
c. …15 or 20…
d. …30 or 40…
e. …60 or 80…
f. …500 or 600...

(12) a. #There were 12 or 10 people at the public meeting.
b. #...10 or 13…
c. #…15 or 25…
d. #…30 or 50…
e. #…18 or 21…
f. #…500 or 502…

For *between* sentences in particular, these restrictions do not hold (though the *between* construction has its own requirement that the gap between the two values be greater than 1).

(13) a. #There were between 12 and 10 people at the public meeting.
b. ?…between 5 and 6…
c. …between 10 and 12…
d. …between 15 and 20…
e. …between 30 and 40…
f. …between 60 and 80…
g. …between 500 and 600…
h. …between 10 and 13…
i. …between 18 and 21…
j. …between 15 and 25…
k. …between 30 and 50 …
l. …between 500 and 502…

Pollmann & Jansen (1996), who examine English ANPs and their counterparts in French, German and Dutch, propose a series of rules for well-formed approximating pairs cross-linguistically. On one way of formulating these rules (due to Eriksson et al. 2010), they are the following:

(14) Rules for well-formed approximating number pairs:
i. the two numbers must be in ascending order;
ii. the gap between them must be a divisor of both values;
iii. the gap must be a so-called favored number, being of the form {1/2/2.5/5}*10;
iv. the gap must be at least 5% of the second value.

Put another way, the felicitous examples involve two successive values in an arithmetic sequence whose starting value and difference value are a favored number of the form in (14iii); furthermore, only values up to the nineteenth/twentieth in the sequence are possible. All of the felicitous cases in (11) conform to these rules. Turning to the infelicitous examples, (12a) is excluded by rule (i). (12b) is excluded by both rules (ii) and (iii), while (12c,d) are excluded by rule (ii). Example (12e) conforms to (ii) but violates the favored number rule (iii). Finally, the infelicity of (12f) is explained as a violation of rule (iv).

Pollmann & Jansen support their proposal with corpus data from the four languages in question, which show that the above restrictions hold in the vast majority of cases (>90% of tokens). Eriksson et al. (2010) provide experimental data from Swedish and American English that are likewise consistent with these rules. The above authors do not, however, compare ANPs to other sorts of two-number approximating expressions (e.g. *between* constructions). We have thus conducted a corpus study to further substantiate that the restrictions described in (14) are specific to ANPs, and not characteristics of range-based expressions more generally.

Data were sourced from the Corpus of Contemporary American English (COCA; Davies 2008-). Using COCA's part of speech tagging, a random sample of 500 tokens was extracted for the sequences *n or m* and *between n and m* (where in each case *n* and *m* were tagged as numerical). After removing irrelevant examples (e.g. disjoined telephone numbers) as well as cases involving a non-decimal numerical base (e.g. time in hours and minutes, height in feet and inches), the data sets were classified on two measures: i) the mathematical properties of the gap between the two numbers; ii) whether or not the gap was a divisor of both numbers. Representative corpus examples are shown in (15) and (16), and the full numerical results are given in Table 1.

(15) a. Who wants to spend five or six hours on a crowded golf course …
b. As we waited, 10 or 12 other people renewing passports came and went.
c. I can envision 30 or 40 people riding around in limos not doing a damn thing …
d. Rose, that pretty blonde girl who came to our house way back in 1935 or 1936 …

(16) a. The soil samples were taken from a depth between 10 and 20 inches …
b. Atlas predicts that between 390 and 425 hotels will be sold in California …
c. In contrast, most areas in northern California yielded between 91 and 113 kg/h …
d. Between 1919 and 1976, 28 bowhead whales were killed or struck…

As seen in Table 1, the posited rules in (14) were again largely confirmed, and the difference between ANPs and *between* constructions was likewise substantiated. Specifically, in the vast majority of ANP tokens, the gap was a 'favored number' in the sense of rule (iii), and was furthermore a divisor of both *n* and *m*, per rule (ii). This was not the case with *between* tokens, roughly two thirds of which featured a non-favored number as gap, with a comparable proportion having a gap that was not a divisor of both values. Chi-square tests show both differences to be significant at $p<0.001$ (favored number: $\chi^2=385.9$; divisor: $\chi^2=420.3$). Note also that this difference is not due entirely to cases of gap equal to 1, which trivially satisfy both rules, and which predominate in the case of ANPs but are uncommon for *between* sentences; the difference remains when we exclude these tokens (favored number: $\chi^2=92.9$; divisor: $\chi^2=100.6$).

A second and more subtle difference between ANPs and *between* sentences is that the former have an approximating or estimating feel that is absent from the latter. Consider the examples in (17). The ANP example (17a) conveys an approximate or 'fuzzy' range ('somewhere between roughly 50 and roughly 60'), and implies that the speaker is uncertain of the exact number. The corresponding *between* example (17b), by contrast, conveys a precisely bounded range; it suggests speaker certainty as to the upper and lower bounds of the range, and is compatible with the speaker knowing (but choosing not to convey) the exact value (cf. Nouwen 2010 for a similar

Table 1. Properties of Gap

	ANP	*between*
Favored number (total)	462	175
1	371	24
10	39	41
100	6	4
$1*10^n$ (n>2)	10	3
2	16	30
20	2	17
200	-	3
$2*10^n$ (n>2)	-	5
5	15	33
50	2	5
500	-	2
$5*10^n$ (n>2)	1	1
25	-	6
250	-	-
$2.5*10^n$ (n>2)	-	1
Non-favored number	17	304
TOTAL	479	479
% Favored number		
- total	96%	37%
- excluding gap=1	84%	33%
% Gap a divisor of both numbers		
- total	96%	32%
- excluding gap=1	81%	28%

point). Perhaps more significantly, there appears to be a truth-conditional difference: in the case that 49 people attended, (17b) is without doubt false, but (17a) seems at least marginally acceptable.

(17) a. There were 50 or 60 people at the public meeting.
b. There were between 50 and 60 people at the public meeting.

Given that these judgments are quite subtle, these patterns were tested via a small online study executed via Amazon Mechanical Turk. Participants saw one of the two brief scenarios in (18), featuring an ANP or a *between* sentence. They were then asked one of two questions, using a 7-point Likert scale. Question 1: Participants were presented with the fact that the actual number was one less than the lower bound of the stated range (i.e. 39 tickets in (18a), 19 newspapers in (18b)), and rated the accuracy of the speaker's statement (7=completely accurate; 1=not at all accurate). Question 2: Participants rated the speaker's certainty (7=completely certain; 1=not at all certain).

(18) a. John and Fred are organizing a charity event for which they are selling tickets to raise money. John, who is responsible for the budget, asks Fred how many tickets have been sold.

Fred answers: We have sold 40 or 50 / between 40 and 50 tickets.

b. At the weekend Luke and Harry usually deliver newspapers to earn some more pocket money. Today is a very cold and rainy day and Luke can't wait to ride back home. Impatiently he asks Harry how many newspapers they still have to deliver.

Harry answers: We still have to deliver 20 or 30 / between 20 and 30 newspapers.

Each participant saw only one scenario in one version (ANP or *between*) and answered one of the two questions, and received $0.05 for participation. Sample size was *n*=25 per scenario/ version/question. The survey was limited to participants with U.S. IP addresses, and a further question confirmed that all participants were native English speakers.

In the analysis, results for the two scenarios were combined, and mean ratings were calculated. The results were the following:

(19) Ratings of range-based expressions:

	Mean		
	ANP	*between*	
Acceptability in '1-under' situation	3.8	2.6	$z = 3.25, p < 0.01$
Certainty	3.7	5.0	$z = -4.02, p < 0.001$

As seen here, the above-stated intuitions are substantiated: ANPs are more acceptable than *between* sentences for values slightly outside of the stated numerical range, while *between* sentences suggest a higher degree of certainty than do ANPs.

These, then, are the facts that need to be explained by a theoretical account of approximating number pairs. In the next section I consider two existing accounts of numerical imprecision from the perspective of whether they are able to do so.

2 Theories of (Im)precision

We would like our account of ANPs to be framed within a more general theory of numerical (im)precision. Before reviewing specific options that are available, it is appropriate to briefly consider what the desiderata for such a theory are. First and most basically, it must provide an account – ideally, a compositional one – for how approximate interpretations are derived for expressions with (apparently) precise denotations. Beyond this, it should be broad enough to encompass both the overt specification of precision level via precision regulators, as in (20) as well as the potential for approximate interpretation of certain bare numerical forms, per (21).

(20) a. Mabel owns about one hundred sheep.
b. Jane arrived at approximately three o'clock.
c. The meeting lasted roughly forty-five minutes.
d. The rope is exactly fifty meters long.

(21) a. Mabel owns one hundred sheep. **(potentially) approximate**
b. Jane arrived at three o'clock. "
c. The meeting lasted forty-five minutes. "
d. The rope is fifty meters long. "

Such a theory should also have something to say about the distribution of imprecise or approximate interpretations. One aspect of this is explaining the difference between round numbers such as in (20), which can be read approximately, and non-round values such as those in (21), which have only precise interpretations (Krifka 2009). Another is to account for the differential availability of approximate interpretations across syntactic contexts, an issue that will be illustrated below.

(22) a. Mabel owns ninety-nine sheep. **precise only**
b. Jane arrived at three-oh-one. ″
c. The meeting lasted forty-three minutes. ″
d. The rope is fifty-one meters long. ″

To be complete, we also should expect a theory of imprecision to account for distributional or interpretational differences among precision regulators. I will not address this aspect of the topic here, but refer the reader to Sauerland & Stateva 2007; Greenberg & Ronen 2013; Bochnak & Csipak 2014 for discussion. Finally, and at a somewhat more philosophical level, a fully satisfactory theory must adequately capture the nature of approximate interpretation. In particular, this means taking a principled and empirically motivated stance on whether imprecision or approximation of interpretation is a semantic or pragmatic phenomenon.

In recent years, two leading approaches to the semantics and pragmatics of imprecision have emerged. In what follows, I briefly review each in light of the above desiderata, and in particular with reference to the data on ANPs and related constructions.

1.1 Pragmatic Halos

Lasersohn (1999) proposes a pragmatic theory of imprecision according to which speakers may utter sentences that, while strictly speaking false, are ‘close enough’ to true for practical purposes. An example is the assertion that *The townspeople are asleep* when the vast majority are sleeping but a small number are, exceptionally, awake.

Closeness to the truth is modeled via **pragmatic halos**. Each expression of the language is assigned a denotation as usual, but this denotation is also associated with a set of entities of the same semantic type – its halo – which represent the values that differ from the denotation in only pragmatically ignorable ways. For example, *three o’clock* denotes some point in time *i*; its halo is a set of times that in the given context are not meaningfully different from *i*.

(23) a. ⟦*three o’clock*⟧ = *i*
b. H_C(⟦*three o’clock*⟧) = {…, *g*, *h*, *i*, *j*, *k*, …}

Halos of complex expressions are derived compositionally from the halos of their constituents. A sentence may then be felicitously uttered if some element in its halo is true, even if it itself is not.

Precision regulators can also be analyzed via halos. Lasersohn proposes that those that specify a high level of precision, such as *exactly*, operate by reducing the halo of an expression to the minimum possible in the context. Those such as *about* that signal a low precision level have the not entirely parallel function of expanding the denotation of an expression to include the halo of the unmodified form.

Considering this proposal in light of the points discussed above, it can be seen that the theory of pragmatic halos achieves the most basic goal of providing a compositional mechanism for deriving the approximate interpretations for both modified and unmodified numerical expressions. Furthermore, while Lasersohn does not consider ANPs, his account can be readily extended to this case as well. Specifically, we may take the pragmatic halo of *forty or fifty* to be compositionally derived from the halos of *forty* and *fifty* via disjunction, interpreted as set union. Then *there were forty or fifty people at the party* may be felicitously uttered if the number *n* who attended was in the halo of either *forty* or *fifty*, which in turn is the case if that number is different from 40 or from 50 in only pragmatically ignorable ways.

There are, however, questions left open by the theory in the form described above. First, it does not offer an explanation for which numerical expressions allow approximate interpretations, and in particular for the difference between round and non-round numbers. By way of example, (24a) could in some contexts be used felicitously to describe an arrival event that occurred at 3:01, but (24b) cannot be used to describe an arrival at 3:00.

(24) a. Jane arrived at three o'clock.
b. Jane arrived at three-oh-one.

This is somewhat surprising: we would expect 'different in only pragmatically ignorable ways' to be a symmetric relation, meaning that if 3:01 is in the halo of 3:00, then 3:00 should likewise be in the halo of 3:01. Apparently, halo size must depend not only on context but also on the form of a numerical expression or perhaps the mathematical properties of its denotation; it is not immediately obvious how this might be achieved. Note that this issue extends to ANPs, in that we have no account for why only certain numerical disjunctions can be interpreted as approximations (cf. (11) vs. (12)).

A second issue faced by the above account is that numerical expressions that allow approximate interpretations in their bare forms fail to do so in certain more complex constructions. Comparative constructions are an example (see Solt 2014 for further discussion), as are *between* sentences. In the appropriate context, *fifty people* in (25a) can be interpreted imprecisely, such that the sentence might be felicitously used in the case that there were in fact exactly 48 people present. But in the same context, (25b) is infelicitous if 49 people attended, even though 49 is greater than a value that would have 'counted as' as *fifty*. The *between* example in (25c) would likewise be infelicitous (cf. the experimental results in Section 2).

(25) a. There were fifty people at the public meeting.
b. There were more than fifty people at the public meeting.
c. There were between fifty and sixty people at the public meeting.

Minimally, then, we require some further mechanism for constraining the availability of approximate interpretations, both in the numerical forms that allow them and the syntactic contexts in which they may arise.

A final point to be aware of is that the pragmatic halos theory – as the name implies – treats the imprecise interpretation of expressions such as *fifty* or *three o'clock* as a pragmatic phenomenon. This commits us to analyzing a large proportion of what speakers say using numerical expressions as strictly false. Events rarely if ever occur exactly at points of time such as 3:00 on the dot; probably no rope in the world is 50 meters long without a deviation of even a few millimeters; and so forth. If one is not prepared to accept this philosophical commitment, halos might instead be implemented as part of the compositional semantics, making imprecision truth conditional in nature. Such a move is made by Morzycki (2011), whose proposal I discuss in greater detail below. But the other issues with the halo-based approach, such as the lack of distinction between round and non-round numbers, remain.

1.1 Scale Granularity

Krifka (2007, 2009) develops an alternate analysis of imprecision based on the **granularity** of measurement scales. The central idea is that the results of measurement can be reported with respect to scales that differ in how coarse- or fine-grained they are, that is, in the size of their minimal units. Distance, for example, might be measured in terms of scales counting in kilometers, tens of kilometers, hundreds of kilometers, and so forth; number might be counted in units, tens, hundreds, etc. Coarser-grained scales have fewer values for representing measurements, and each of these values stands for a broader scalar range than values on a finer-grained scale. Thus the coarser the scale on which a numerical expression occurs, the more approximate interpretation it allows.

Krifka proposes that in our culture, scales are most commonly based on powers of 10, or the result of applying operations of halving or doubling to such scales. As an example, (26) displays three possible scales for the measurement of distances. Speaking more precisely, i.e. relative to the finest of these scales (26a), the distance indicated by the arrow would be described as *forty-six meters*. But speaking more approximately, this same distance might be described as *forty-five meters* (scale (26b)) or even *fifty meters* (scale (26c)).

(26) Scales for distance measurement:

```
a. …–39m–40m–41m–42m–43m–44m–45m–46m–47m–48m–49m–50m–51m–52m–…
b. …-----40m----------------45m----------------50m---------…
c. …-----40m-------------------------------------50m---------…
                                 ⬆
```

While scale structures such as those in (26) derive from the structure of the decimal number system, in some domains the available granularity levels are based on other principles. Such is the case in time measurement, where the convention of dividing the day into 60-minute hours results in scales counting in units of 30 minutes, 15 minutes and 5 minutes. The above-discussed approximate interpretation of *three o'clock* thus might involve a scale of 5- or 15-minute granularity.

On the granularity-based approach, round numerical expressions are ambiguous between (more) precise and (more) approximate interpretations. A further component of Krifka's theory is thus a pragmatic Principle of Strategic Interpretation based on Parikh (1991), which specifies that the preferred interpretation is the one that has the greatest likelihood of being true. For simple numerical expressions such as those in (20), this favors interpretation relative to the coarsest-grained scale on which the expression has an interpretation, as this is compatible with the most possible situations. The result is a preference for – rather than simply the availability of – the approximate interpretation.

Sauerland & Stateva (2009) show that a granularity-based approach also allows a natural account of the semantics of precision regulators. Granularity on their account is a contextual parameter of interpretation. Precision regulators can then be analyzed as resetting the level of granularity at which a numerical expression is interpreted: those such as *exactly* set the granularity parameter to the finest available in the context, whereas those such as *approximately* set it to the coarsest level.

The granularity-based theory has an advantage over the theory of pragmatic halos, in that it not only provides a mechanism for generating approximate interpretations, but also gives an account of which numerical expressions allow these. Approximate interpretation is interpretation relative to a coarse-grained scale. Round numbers occur on coarser-grained scales than do non-round numbers, and thus allow more imprecise interpretations. This same mechanism gives us a way to understand the restrictions on ANPs. Specifically, the felicitous examples are those involving the disjunction of two sequential values on a scale of some allowable granularity level. The interpretation that obtains is correct as well: *fifty or sixty people* is some number of people that, at granularity level 10, would be described as either *fifty* or *sixty*; this includes values between 50 and 60, but also those slightly outside of this range (cf. the experimental results from Section 2).

The theory presented above, however, faces the same issue as the theory of pragmatic halos in accounting for constructions in which round numbers disallow an approximate interpretation. Consider again an example such as *between fifty and sixty people*. If *fifty* and *sixty* are interpreted relative to a coarse-grained scale (say, a scale of granularity 10), the resulting scalar range is broader than that which obtains if they are interpreted precisely. By the above-described pragmatic principle, the approximate interpretation should then be favored. But this is at odds with the earlier observation (and our experimental findings) that numerical values in *between* sentences are interpreted precisely.

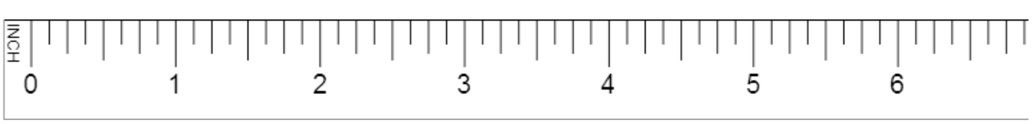
Finally, analyzing imprecision in terms of scale granularity gives us a route to avoiding the proliferation of falsity that arises when imprecision is analyzed via pragmatic halos. If we take coarse-grained scales to be part of the semantics, then an expression such as *three o'clock* has an interpretation on which it denotes a range around the point 3:00; as such, *Jane arrived at three o'clock* can be true in a situation where her actual arrival time was 3:01. But this solution too might cause us a bit of discomfort: it seems that we would want *three o'clock* at the most basic level to denote a single point in time, and even more clearly *one hundred* to denote the single value 100. Thus this alternative is also not entirely satisfying from an intuitive perspective.

In summary, neither of the two leading theories fully achieves the desiderata outlined at the start of this section, nor captures all of the data of interest in the present work. In the next section, I present a revised approach that I hope takes us somewhat closer to these goals.

3 The Ruler Model of Granularity

In this section, I outline a new implementation of the granularity-based approach, originally proposed in Solt (2014), which seeks to overcome the issues with existing theories of imprecision that were discussed above. The starting point for the proposal is the metaphor of the ruler as measuring instrument:

(27)



A ruler is itself a continuous extent, but a discrete structure is imposed on it via a hierarchical system of markings of different degrees of prominence, which make it possible to perform measurement at varying levels of precision. In the example above, for instance, we could choose to measure lengths in inches, in half inches, in quarter inches, or in eighth inches. In each case, the measurement that will be reported is that corresponding to the closest mark at the chosen level. I propose that this gives us a clue as to the correct view of granularity as it pertains to natural language measure expressions. The proposal consists of several steps, which I outline below.

i) Precise denotations for measure expressions. I assume a semantic framework that includes degrees as a basic type (type *d*). Degrees may be conceptualized as points on a maximally fine-grained scale. In the case of cardinality, the relevant scale is the positive integers, or perhaps the rational or real numbers (see Fox & Hackl 2006); in other cases, it is a dense scale associated with some other dimension of measurement. I then take measure expressions to denote quantifiers over degrees (type $\langle\langle dt\rangle, t\rangle$), which can be lowered to a term meaning (type *d*) via generally applicable type-shifting operations (Kennedy 2015):

(28) a. $[\![fifty_{\langle\langle dt\rangle,t\rangle}]\!] = \lambda I_{\langle dt\rangle}.max(I)=50$
b. $[\![fifty_d]\!] = 50$

(29) a. $[\![twenty\ meters_{\langle\langle dt\rangle,t\rangle}]\!] = \lambda I_{\langle dt\rangle}.max(I)=20$ m
b. $[\![twenty\ meters_d]\!] = 20$ m

This contrasts with previous granularity-based approaches in which measure expressions have interpretations on which they denote scalar ranges.

ii) Granularity as alternatives. Granularity may now be represented in terms of sets of alternatives to a measure expression. We start with some standard unit *gran*, which defines a standard sequence S_{gran}. In (30)-(32), I give examples of standard sequences for the measurement of number, length and duration, respectively:

(30) a. *gran* = 10 S_{10} = {10, 20, 30, 40, …}
b. *gran* = 20 S_{20} = {20, 40, 60, 80, …}

(31) a. *gran* = 5 m S_{5m} = {5 m, 10 m, 15 m, 20 m, …}
b. *gran* = 100 m S_{100m} = {100 m, 200 m, 300 m, 400 m, …}

(32) a. *gran* = 5 min S_{5min} = {5 min, 10 min, 15 min, 20 min, …}
b. *gran* = 15 min S_{15min} = {15 min, 30 min, 45 min, 60 min, ...}

As discussed earlier, typical choices for *gran* are powers of 10 and the results of halving and doubling these. In some cases, domain specific measurement conventions also come into play, as in (32b), which is based on the convention of dividing a 60-minute hour into halves and quarters.

For a measure expression *α* and granularity level *gran*, a set of alternatives to *α* can then be defined as in (33); examples are given in (34).

(33) $ALT_{gran}(\alpha) = \{\alpha' : [\![\alpha']\!] \in S_{gran}\}$

(34) a. $ALT_{20}(sixty) = \{. . . , forty, sixty, eighty, . . . \}$
b. ALT_{5m}(*fifty meters*) = {. . . , *forty-five meters*, *fifty meters*, *fifty-five meters*, . . . }

We take $ALT_{gran}(\alpha)$ to be undefined if $[\![\alpha]\!]$ is not in S_{gran}; in other words, a measure expression only has alternatives at granularity levels in which its denotation participates.

The members of ALT_{gran} can be likened to the markings on the ruler: they represent the (only) choices the speaker has to report a measurement at granularity level *gran*.

iii) Truth relative to granularity level. We come now to the crucial step. Following Sauerland & Stateva (2007), I take the truth or falsity of a sentence containing a measure expression to be relativized to the granularity level at which that expression is interpreted. For each measure expression, the corresponding granularity level *gran* is contextually determined via an assignment function *g*. I then propose that truth at a granularity level is defined in terms of the scalar distance that the actual exact measure would need to be displaced in order to achieve truth under a perfectly precise interpretation, which we could express as interpretation relative to the minimum possible granularity level *gran=min* (in the case of cardinality measurement, i.e. counting discrete entities, this is *gran*=1; in other cases it is some hypothetical minimum granularity level). Formally:

(35) For a sentence φ containing a measure expression α, $[\![\varphi]\!]^g = 1$ iff there is no $\alpha' \in ALT_{gran}(\alpha)$ such that $[\![\varphi[\alpha' / \alpha]]\!]^{g[gran=min]} = 1$ would require a smaller scalar displacement of the actual measure than $[\![\varphi]\!]^{g[gran=min]} = 1$.

In simple terms, a sentence containing a measure expression is thus evaluated as true iff there is no better choice of expression at the relevant granularity level.

Before returning to the topic of range-based approximations, let us briefly observe how the above components yield the desired results for simpler examples. Consider the sentences in (36) and (37), which I assume to have the truth conditions indicated.[1]

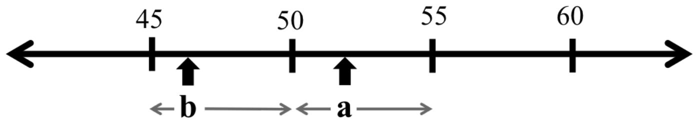
(36) There were fifty people at the meeting.
max{*d*: there were *d* people at the meeting} = 50

(37) There were more than fifty people at the meeting.
max{*d*: there were *d* people at the meeting} > 50

[1] The representation in (36) is based on the degree-quantifier interpretation of the number word; here I make a standard assumption that this originates DP internally, but QRs for type-related reasons, taking as argument the set of degrees formed by lambda abstraction over a type *d* trace in its base position (Heim 2000, Nouwen 2010). Note that this derives a double-bounded 'exactly' reading for the sentence as a whole. A lower-bounded 'at least' reading can be generated via the type *d* interpretation of the number word, per Kennedy 2015. The representation in (37) can be derived via standard treatments of the comparative (see e.g. Beck 2011), in the most simple implementation involving the comparative morpheme *-er* taking the number word in its type *d* interpretation as argument, producing a quantifier over degrees.

For purposes of concreteness, we will assume that the granularity level assigned by *g* to the measure expression *fifty* is *gran* = 5, and consider the two situations depicted in (38):

(38)



Starting with (36), the replacement of *fifty* with members of ALT_5(*fifty*) yields sentences including the following to be evaluated at *gran*=*min* (i.e. *gran*=1):

(39) a. ⟦*there were forty-five people at the meeting*⟧$^{g[gran=min]}$
b. ⟦*there were fifty people at the meeting*⟧$^{g[gran=min]}$
c. ⟦*there were fifty-five people at the meeting*⟧$^{g[gran=min]}$

If the actual number of attendees is as in **(a)** in (38), the original (36) will be evaluated as true, because (39b) is the alternative whose truth would require the smallest scalar displacement of this measure. But if the actual number is **(b)**, the sentence will be evaluated as false, since there is a better expression choice at this granularity level: the truth of (39a) requires a smaller displacement of the actual measure.

Crucially, a different pattern obtains for the comparative in (37). In this case, the sentential alternatives that are derived include the following:

(40) a. ⟦*there were more than forty-five people at the meeting*⟧$^{g[gran=min]}$
b. ⟦*there were more than fifty people at the meeting*⟧$^{g[gran=min]}$
c. ⟦*there were more than fifty-five people at the meeting*⟧$^{g[gran=min]}$

In situation **(a)**, or any other case in which the actual number is greater than exactly 50, (37) will be evaluated as true, because (40b) is already true at *gran*=*min* with no displacement of the actual value needed. Conversely, it will be evaluated as false in situation **(b)**, or any other case of actual number ≤50, because there are other alternatives for which it is already true at *gran*=*min*; in situation (b), for example, (40a) is such an alternative. Furthermore, this is the case regardless of how *gran* is set contextually: (37) is evaluated as true in all and only those cases that the actual number is greater than (exactly) 50, and false otherwise. The result is a necessarily precise interpretation for the numerical expression.

Relative to the desiderata outlined at the start of the previous section, the present ruler-inspired model maintains the empirical coverage of previous granularity-based approaches, while also offering some potential advantages. Specifically, we are able to account for not only the interpretational difference between round and non-round numbers, but also the restrictions on when the former can be interpreted imprecisely. The general picture is the following. Via the interpretation rule in (35), measure expressions with point-based denotations (e.g. *fifty*) receive approximate interpretations. This has the effect of filling in the scalar territory in between the points in S_{gran}, allowing any measured value to be truthfully described by some member of ALT_{gran}. But the same values embedded in range-denoting expressions (e.g. *more than fifty*) are interpreted precisely, because no 'filling in' is necessary to allow any measurement value to be described, whatever the prevailing granularity level is. Below we will see that this distinction is what is needed to account for the facts relating to ANPs and *between* constructions. On a more philosophical note, the present theory takes a stand on the semantic versus pragmatic nature of imprecision that avoids certain of the more unintuitive aspects of previous accounts. Number words always have point-based denotations; but sentences in which they occur may nonetheless be evaluated as true (and not just 'close enough to true') if the actual measure deviates from that point.

Before proceeding, it is appropriate to briefly relate the proposal outlined above to another theory that makes use of alternatives in the analysis of imprecision, namely that of Morzycki (2011). In the context of developing an analysis of metalinguistic comparatives, Morzycki proposes a compositional implementation of Lasersohn's pragmatic halos in which clusters of similar meanings (i.e. halos) are represented as Hamblin alternatives. The interpretation function is then relativized to a degree of precision that determines the size of this halo. A comparative such as *George is more dumb than crazy* can then be interpreted to mean that there is a degree of precision at which the halo around *dumb* contains a predicate true of George while the halo around *crazy* does not. On one level, this is quite different from the present approach. In the proposal developed here, the alternatives that play a role in imprecise interpretation are alternatives to the measure expression itself, while in Morzycki's account, they are alternatives to an expression's precise denotation. At a deeper level, there is however an important connection between the two. Specifically, both Morzycki's approach and the present one are based a mechanism whereby an expression with a precise denotation may be used truthfully when the precise case does not obtain. In light of this, I suspect that it might be possible to integrate the two approaches into a more general account of imprecision, encompassing both numerical and non-numerical cases; but I do not attempt this here.

4 Approximating Number Pairs and their Relatives

In this section, the model outlined above will be applied to approximating number pairs and *between* sentences. The analytical insight that forms the basis of the proposal is that the former involve the disjunction of two point-denoting expressions, while the latter are inherently range-denoting expressions.

Formally, I take the denotations of numerical disjunctions to be derived from the degree-quantifier interpretations of the component numerical expressions, with *or* interpreted as set union (per Keenan & Faltz 1985, among others):

(41) $[\![\textit{forty or fifty}]\!] = [\![\textit{forty}]\!] \cup [\![\textit{fifty}]\!]$
$= \lambda I_{<dt>}.max(I)=40 \cup \lambda I_{<dt>}.max(I)=50$
$= \lambda I_{<dt>}.max(I)=40 \text{ or } 50$

Between, on the other hand, takes as arguments two numerical expressions on their degree-denoting interpretations, and forms from them a range-denoting quantifier over degrees:

(42) a. $[\![\textit{between}]\!] = \lambda d \lambda d' \lambda I_{<dt>}.d \leq max(I) \leq d'$
b. $[\![\textit{between forty and fifty}]\!] = \lambda I_{<dt>}.40 \leq max(I) \leq 50$

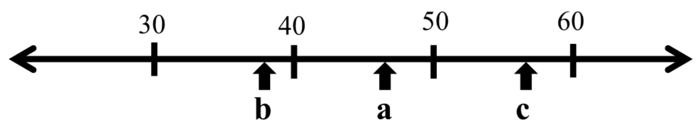
We thus derive the following interpretations for two relevant examples:

(43) There were forty or fifty people at the meeting.
max{*d*: there were *d* people at the meeting} = 40 or 50

(44) There were between forty and fifty people at the meeting
40 ≤ *max*{*d*: there were *d* people at the meeting} ≤ 50

Let us now observe how the evaluation of the above expressions is affected by the granularity-based interpretation rule in (35). We begin with the disjunctive case (43), where we assume to start that the granularity level is set to *gran*=10. For concreteness, we may consider the situations depicted in (45).

(45)



By the same logic that applied in the case of a simple numerical expression (e.g. *fifty*), (43) can be truthfully uttered when either *forty* or *fifty* is at least as good a description of the true number as any other member of ALT_{10}(*forty*) (which, it should be noted, is identical to ALT_{10}(*fifty*)). This is the case for all values between 40 and 50 (inclusive), such as **(a)** in (45). It is also the case for values outside of this range, as long as they are at least as close to 40 or 50 as to any other value at this granularity level, i.e. any other multiple of 10; this holds for **(b)** in (45), but not for **(c)**. In this way, *forty or fifty* comes to convey a range with fuzzy boundaries.

Crucially, that a range interpretation is derived at all is due to the choice of granularity level *gran* as equal to the gap between the two conjoined numerals (10 in the case under consideration), as this has the effect of 'filling in' the scalar territory between the two values. I propose that this is precisely the situation in which the approximative interpretation arises. I believe this is based on a reasonable assumption: in choosing an expression such as *forty or fifty*, the speaker is signaling that the context is such that we are counting individuals in units of (in this case) 10. The interpretation that then arises is that either *forty* or *fifty* is the best choice at *gran*=10, which in turn means that the true value is somewhere between roughly 40 and roughly 50. In this way, we are able to account for the primary restrictions on which pairs can form ANPs (see Section 2): the gap must be an acceptable choice for *gran*, and as standard sequences are formed as multiples of *gran* (i.e. {1**gran*, 2**gran*, 3**gran*, …}), the gap must likewise be a divisor of both values.

Note also that nothing prevents a numerical disjunction such as (43) from being interpreted at a finer granularity level, in particular at *gran*=*min*. In this case a 'true disjunction' reading obtains. This reading is easier to get with overt *either*, e.g. *either forty or fifty people*, though I believe it is available for the 'bare' disjunction as well. Furthermore, such an interpretation is available regardless of the round vs. non-round character of the numerals involved or the mathematical properties of the gap.

A different picture emerges for *between* sentences such as (44). By the same logic that applies to comparatives such as *more than fifty*, (44) will regardless of the choice of *gran* be evaluated as true for all values between 40 and 50 inclusive (e.g. **(a)** in (45)), because it is already true for such values on a perfectly precise interpretation, with no need for displacement of the actual measure. Conversely, it will be interpreted as false for all values outside of this range (e.g. **(b)** and **(c)** in (45)), because however *gran* is set, there are other expression choices in

ALT_{gran} that would yield a true statement at a perfectly precise interpretation. Again assuming *gran*=10, for example, the measure **(b)** in (45) could be described as *between thirty and forty.* The result is that *between* sentences – unlike ANPs – are interpreted as conveying ranges with sharp boundaries. Furthermore, since the range interpretation is lexicalized by *between* itself rather than arising via the interaction of the component values and the contextually determined granularity level, there are no restrictions on which pairs can occur in *between* sentences.

To summarize, with the proposed approach to numerical imprecision as a whole, we are able to account for some puzzling characteristics of ANPs, as well as their differences versus an apparently similar sort of range-based approximating expression.

5 Conclusions and Possible Extensions

There is a growing body of literature on the semantics and pragmatics of approximate or imprecisely interpreted numerical expressions. The present work continues this line of research, focusing on a lesser-studied and initially puzzling means of expressing approximations, namely approximating number pairs. These have the curious property that they are disjunctive in form, but convey not disjunctions but rather approximate ranges. I have suggested that the existence of such constructions, and the constraints on their formation, provide support for a granularity-based analysis of numerical imprecision. Indeed, such pairs are some of the strongest evidence we have that granularity is not merely a formal construct, but rather something that speakers represent and make use of: we can count by twos, fives, tens, fifties, hundreds and so forth (but not, for instance, by threes, sevens or nineteens); in doing so, we are able to describe measurement results in increasingly coarse-grained terms. In this paper, which builds on Solt (2014), I have proposed a novel implementation of the granularity-based approach in which granularity is construed as sets of alternatives. Augmented with an interpretation rule that allows a sentence with a measure expression to be used when that expression is the best choice at the contextually determined granularity level, this approach has allowed us to provide a compositional analysis of approximating number pairs, while also accounting for their differences relative to other range-based expressions, and other facts relating to numerical imprecision.

In concluding, I would like to briefly discuss the outlook for extending this approach beyond the data considered here. An initial area for exploration concerns the counterparts of approximating number pairs in languages other than English. Here, the first crucial point to recall is that these sometimes are formed with disjunction as in English (e.g. Spanish *diez o veinte personas* 'ten or twenty people'), but sometimes by simple juxtaposition of the two values (e.g. German *zwei, drei Tage* 'two or three days'; *fünfzig, sechzig Meter* 'fifty or sixty meters'). Given this pattern, if we are to seek a unified analysis, it is natural to ask if it is really possible to maintain the view that the *or* of English ANPs is ordinary disjunction, as I have assumed here. This doubt is reinforced by the observation that sentential disjunction (e.g. *There were fifty people at the meeting or there were sixty people there*) does not produce the approximating effect that is observed with ANPs. I think this is a valid question, but I am not aware of any alternative mechanism that could be substituted for disjunction, and in any case the disjunction-based analysis nicely captures the facts of ANPs. I believe, therefore, that it is just as plausible to analyze examples from languages such as German that lack an overt 'or' to be covert disjunctions. As for examples featuring sentential disjunction, I hypothesize that the difference in these cases is that by separating the two numerical values, it is no longer implied (as it is in the ANP case) that the relevant granularity level is that of the gap between the two values; as we saw above, if *gran* is set to some other value (e.g. *gran*=1), the result is a true disjunction reading.

A further area to investigate involves other sorts of range-based numerical expressions in English and cross-linguistically. As briefly mentioned above, English has at least one more such construction, namely *to* sentences (e.g. (*from*) *fifty to sixty people*). Preliminary corpus analyses show these to pattern with *between* sentences rather than ANPs in terms of the values that may occur in them. There are also, however, differences between the two. In particular, Nouwen (2011) observes that *between* constructions are compatible with a definite value known to the speaker (per (46a)), placing them in the same category as comparative quantifiers such as *more than fifty people*. *To* sentences, particularly in the variant with *from*, are not (per (46b)), aligning them to a category that also includes superlative quantifiers such as *at most fifty people*.

(46) a. My ticket to the concert cost between €100 and €200. Specifically, it cost €135.
b. # My ticket to the concert cost (from) €100 to €200.

We might ask if an account framed in the present terms might have something to say about the source of this difference, though I do not have anything to offer at present.

A final set of data that merits exploration relates to the co-occurrence of approximating number pairs with other forms of precision regulation, as illustrated below:

(47) a. about/approximately/roughly forty or fifty people
b. forty or fifty people or so / or thereabouts

In Solt (2014), I argue that precision regulators such as *about* should be analyzed as functions that map points to scalar ranges, with those ranges having the status of approximate or coarse-grained degrees. The data in (47) suggest that such mapping can also be made from pairs of values; but the details of how this might work compositionally are far from clear. I leave this as a topic to be pursued in the context of a more detailed study of the semantics and pragmatics of precision regulators.

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