Approximators as a case study of attenuating polarity items*

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1. Introduction

Approximators such as *about* and *roughly* exhibit a curious and little-recognized form of polarity sensitivity. As illustrated below, in simple sentences they are positive polarity items (PPIs), whereas when embedded in comparative quantifiers, they become negative polarity items (NPIs):

(1) a. Lisa has about/roughly/approximately 50 sheep.
 b. *Lisa doesn't have about/roughly/approximately 50 sheep.

(2) a. *Lisa has more than about/roughly/approximately 50 sheep.
 b. Lisa doesn't have more than about/roughly/approximately 50 sheep.

As a caveat, (1b) and (2a) do have a use on which they are felicitous, namely when they serve to contradict or deny an assertion involving the approximator in the prior discourse context. If for example someone has previously claimed that Lisa has 'about 50' sheep, I may counter that "no, she doesn't have 'about 50' " or "no, she has more than 'about 50' ". We will put this quotative or echoic usage aside, and focus on the occurrence of sentences such as those in (1)-(2) in a neutral discourse context.

In their felicitous uses, approximator-modified numerical expressions make weaker assertions than salient alternatives, notably their unmodified counterparts. Assuming that *fifty* and *about fifty* describe the scalar ranges depicted in (3), we obtain the asymmetric entailments indicated in (4).

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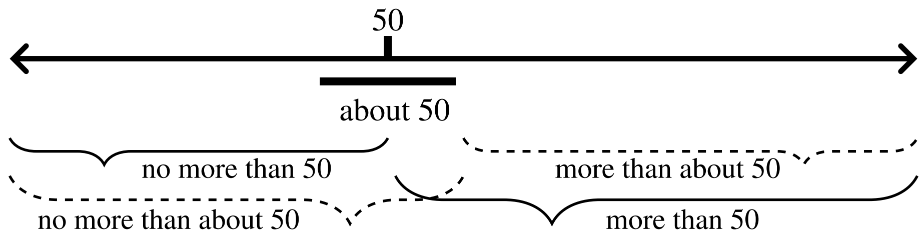
(3) Ranges corresponding to numerical expressions:



(4) a. Lisa has 50 sheep. ⇒ Lisa has about 50 sheep.

b. Lisa doesn't have more than 50 sheep. ⇒ Lisa doesn't have more than about 50 sheep.

On this basis, we can assign approximators to the class of **attenuating polarity items** (Israel 1996, 2006, 2011), whose characterizing feature is that they make relatively weak or understated assertions relative to possible alternatives. Additional members of the class include the NPI *much* (*Homer *slept much / didn't sleep much*) and the PPI *fairly* (*Bart is/ *isn't fairly lazy*).

Despite decades of research on the semantics and pragmatics of polarity items, attenuators remain relatively understudied (notable exceptions are Israel's work and more focused contributions such as Matsui 2011 and Onea & Sailer 2013). With regards to approximators in particular, the PPI status of simple approximator examples is discussed by Rodríguez (2008) and Spector (2014), while Solt (2014) observes that approximators in comparatives pattern as NPIs. Yet to my knowledge the only discussion of the reversal in polarity sensitivity illustrated in (1)-(2) is a brief mention in Israel (2006), and there is no fully worked out existing account that is able to capture these data. The goal of this paper is to formulate such an account, as a step towards a more general theory of attenuating polarity items.

The two central intuitions that form the basis of the proposal developed here are these:

- Approximators, like other polarity items, obligatorily introduce alternatives. Because they are modifiers, their alternatives necessarily include the corresponding unmodified form.
- Being vague, approximators do not readily make stronger statements than salient alternatives – notably the unmodified one. They are thus restricted to situations in which the simpler unmodified form could not have been asserted.

The challenge is to justify these intuitive claims, and develop them into a rigorous theory that accurately accounts for the polarity-based distributional restrictions that characterize approximators – and other members of the attenuating class.

The structure of the paper is as follows: Section 2 develops the neo-Gricean framework in which the present account will be couched. Section 3 applies this framework to the polarity sensitivity of approximators. Section 4 considers in more depth the implicatures that arise with approximators of different sorts in their felicitous uses. Finally, Section 5 contrasts the present approach to one based on a grammatical theory of implicature, and Section 6 concludes.

2. Pragmatic framework

A wide variety of linguistic phenomena have been productively analyzed with reference to alternatives to a linguistic expression that the speaker could have but did not utter. This includes the semantics of focus-sensitive particles, scalar and other sorts of implicatures, and – most relevant to the present topic – polarity sensitivity. I follow this broad tradition here. More specifically, I adopt a neo-Gricean pragmatic approach to the role of alternatives in linguistic meaning, based on proposals by Krifka (1995) and more directly Katzir (2007).

The central component of the formal system I assume is the following conversational principle or rule of assertion from Katzir:

(5) **Conversational principle:** Do not use ϕ if there is another sentence $\phi' \in ALT(\phi)$ such that both:

i. ϕ' is better than ϕ ($\phi' \succ \phi$)   ii. ϕ' is weakly assertable

Before delving into the definitions of the terms in (5), let us first note that this principle has two important consequences:

1. In asserting ϕ, the speaker implicates that all better alternatives ϕ' cannot be asserted.
2. A sentence ϕ is blocked when an implicature derived in this way contradicts the original assertion, or equivalently, when ϕ always has a better alternative.

It is the second of these points that will form the basis of the account of the polarity-based restrictions on approximators developed here.

Let us turn now to how the component parts of (5) should be understood. To start, I adopt Katzir's structural view of alternatives, according to which they are derived via substitution and deletion. The formal details are the following:[1]

(6) **Substitution source:**
Let ϕ be a parse tree. The substitution source for ϕ, written as $L(\phi)$, is the lexicon of the language.

(7) **Structural alternatives:**
Let ϕ be a parse tree. $ALT(\phi)$ – the set of alternatives to ϕ – is the set of parse trees ϕ' that can be derived from ϕ via a finite series of deletions, contractions, and replacements of constituents in ϕ with constituents of the same category taken from $L(\phi)$.

[1]Note that Katzir discusses cases in which the substitution source cannot be equated with the lexicon, per (6), but also includes elements from the prior discourse; as these issues do not come up in the present context, I skip this refinement here.

Also following Katzir, I take a sentence to be 'weakly assertable' if the speaker believes it to be true, relevant and supported by the facts.

The definition of the 'better than' relation $\succ$ is where things become more interesting. Building on a possibility discussed by Katzir (though not the option he ultimately adopts), I propose that – at least in the case of attenuating polarity items – 'better than' should be understood in terms of both informativity and simplicity. This position embodies the often conflicting pressures towards informativity and simplicity embodied in Grice's (1975) maxims of Quantity and Manner, and in Horn's (1984) Q- and R-principles. Formally, I assume the following definition:

(8) $\phi \succ \psi$ iff $\phi \gtrsim_{INF} \psi \wedge \phi \gtrsim_{SIMP} \psi \wedge (\phi \succ_{INF} \psi \vee \phi \succ_{SIMP} \psi)$

In words, (8) states that ϕ is better than ψ if it is at least as informative and at least as simple as ψ, and has an advantage on either informativity or simplicity (or both).

The simplicity relation $\phi \gtrsim_{SIMP} \psi$ can in turn be defined in structural terms:

(9) **Simplicity**:

a. $\phi \gtrsim_{SIMP} \psi$ iff ϕ can be derived from ψ via substitution/deletion

b. $\phi \succ_{SIMP} \psi$ iff $\phi \gtrsim_{SIMP} \psi$ and not $\psi \gtrsim_{SIMP} \phi$

That is, ϕ is simpler than ψ if ϕ can be derived from ψ via a series of substitutions or deletions, but not vice versa.

The informativity relation $\phi \gtrsim_{INF} \psi$ turns out to be more complicated. In most work in the alternatives-based tradition, informativity is understood in terms of asymmetrical entailment: ϕ is more informative than ψ if the set of worlds in which ϕ obtains is a proper subset of the set of worlds in which ψ obtains. This is fully adequate for the paradigm cases to which such theories have been applied. For example, a relation of asymmetric entailment holds between *and* and *or*, between *(at least) 4* and *(at least) 3*, and between *all* and *some*. But it becomes problematic when ϕ or ψ or both are vague (or, more accurately, have constituents that are vague). The issue is that on one way of resolving the vagueness, ϕ might asymmetrically entail ψ, but on another, the two might be equivalent, or it might even be ψ that asymmetrically entails ϕ.

To deal with this issue, I propose that the 'more informative' relation $\phi \gtrsim_{INF} \psi$ must be understood as 'definitively more informative than' (in way that will be made clearer below). There is in fact independent motivation from the domain of scalar implicatures that something like this is necessary. van Tiel et al. (2016) demonstrate that scalar implicatures are more likely to arise between pairs of adjectives when the scalar distance between them is greater (e.g. *difficult* tends to implicate *not impossible*, whereas *content* does not consistently implicate *not happy*). As an even more relevant parallel, Leffel et al. (to appear) observe an asymmetry in the interpretation of *not very Adj*: with minimum-standard absolute gradable adjectives, this licenses the inference that the positive form *Adj* holds, while with context-sensitive relative gradable adjectives, no such inference is drawn:

(10) a. John was not very late. $\rightsquigarrow$ John was late.
b. The antenna is not very bent. $\rightsquigarrow$ The antenna is bent.

(11) a. John is not very tall. $\not\rightsquigarrow$ John is tall.
b. John isn't very smart. $\not\rightsquigarrow$ John is smart.

Put differently, the examples in (10) license the inference to the negation of the simpler form *not Adj*, while the examples in (11) do not. Leffel and colleagues account for this pattern via a constraint on the derivation of implicatures with vague predicates, according to which such implicatures are not drawn if the conjunction of a sentence with its stronger alternative would necessarily be a **borderline contradiction** (Alxatib & Pelletier 2011, Ripley 2011). Intuitively, there are situations that are simultaneously clear cases of both *late* and *not very late*; but any individual that is both *tall* and *not very tall* is necessarily a borderline case of both predicates. Thus the vague standards of *tall* and *very tall* are not sufficiently distinct for an implicature to be drawn from *not very tall* to *NOT(not tall)=tall*.

I retain the insight articulated by Leffel et al. (to appear), though giving it a somewhat different implementation. Formally, I follow Krifka (2012) in taking expressions of language to be interpreted relative to a pair of indices $\langle w,i\rangle$, where w is a world index and i an interpretation index. That is, the usual notion of a world parameter is decomposed into world and interpretation components. If $[\![\alpha]\!]^{w,i} \neq [\![\alpha]\!]^{w',i}$, this means that there is some factual difference in the state of affairs at indices $\langle w,i\rangle$ and $\langle w',i\rangle$. If on the other hand $[\![\alpha]\!]^{w,i} \neq [\![\alpha]\!]^{w,i'}$, the difference lies instead in how expressions of the language are interpreted at indices $\langle w,i\rangle$ and $\langle w,i'\rangle$. The common ground can be modeled as a pair $\langle W,I\rangle$, where W is a set of worlds and I a set of interpretations.

With this framework in place, it is possible to define relative strength or informativity in several distinct ways, which differ in how stringent they are. Starting with the 'at least as strong as' relation in (12), (13)-(15) represent some possibilities:

(12) ϕ is **at least as strong as** ψ ($\phi \succeq_S \psi$) iff $\forall i, \{w : \phi_{i,w} = 1\} \subseteq \{w : \psi_{i,w} = 1\}$

(13) ϕ is **weakly stronger than** ψ ($\phi \succ_{WS} \psi$) iff:

a. $\phi \succeq_S \psi$
b. $\exists w,i[\psi_{w,i} = 1 \wedge \phi_{w,i} = 0]$ (i.e. $\neg\psi \succeq_S \phi$)

(14) ϕ is **strictly stronger than** ψ ($\phi \succ_{SS} \psi$) iff:

a. $\phi \succeq_S \psi$
b. $\forall i \exists w[\psi_{w,i} = 1 \wedge \phi_{w,i} = 0]$

(15) ϕ is **definitively stronger than** ψ ($\phi \succ_{DS} \psi$) iff:

a. $\phi \succeq_S \psi$
b. $\exists w \forall i,i'[\psi_{w,i} = 1 \wedge \phi_{w,i'} = 0]$

The definition in (13) states that ϕ is (weakly) stronger than ψ iff there is some interpretation i on which it is stronger (understood in the usual way in terms of asymmetrical entailment). That in (14) says that ϕ is (strictly) stronger than ψ iff it is stronger on all interpretations. Finally, (15) is even stricter, specifying that ϕ is (definitively) stronger than ψ iff there is some world for which across all possible interpretations of ϕ and of ψ, the latter is true while the former is false.[2]

I would like to argue that for the purpose of selecting between alternatives, the 'more informative' relation $\succ_{INF}$ in (8) must be understood as 'definitively stronger' $\succ_{DS}$, per (15). The rationale is the following: In order for ϕ to be preferable to ψ on grounds of informativity, there must necessarily be some situation or state of affairs that – regardless of how any vagueness in the interpretations of each is resolved – makes ψ true but ϕ false. If this constraint did not hold, the proposition conveyed by an utterance of ϕ on some interpretation i could in fact be that conveyed by ψ on some other equally acceptable choice of interpretation i'; and furthermore, the negation of ϕ relative to some interpretation i could actually contradict ψ on some equally legitimate interpretation i'. Put differently, on this quite stringent definition of greater informativity, there is at least one situation that is definitively allowed by the weaker sentence but ruled out by the stronger one.

In the next section, we will see how the system as elaborated here accounts for the polarity sensitivity of approximators.

3. Approximators and polarity

We begin with some preliminaries. Following Kennedy (2015), I take cardinal numerals to have type-flexible semantics, with denotations as both degrees and quantifiers over degrees; the latter yields a 2-sided 'exact' rather than 'at least' interpretation for the numeral. I further adopt the approach of Krifka (2007) in taking numerical expressions to be interpreted at a contextually determined level of granularity, which accounts for the imprecise interpretations available to round numbers in particular. Formally, this may be modeled via a granularity parameter *gran*, whose value is one of those determined by the interpretation index i (see Sauerland & Stateva 2007 and Solt 2014 for alternate possible formal implementations). Thus we have (16) as the interpretation of a simple numerical example. Assuming a standard analysis of comparative quantifiers as degree quantifiers (see e.g. Nouwen 2010) yields (17) as the interpretation of a comparative example:

(16) Lisa has 50 sheep. $max\{n : \text{Lisa has } n \text{ sheep}\} = 50_{gran^i}$

(17) Lisa has more than 50 sheep. $max\{n : \text{Lisa has } n \text{ sheep}\} > 50_{gran^i}$

Note that this analysis assumes a semantic rather than pragmatic approach to imprecision, according to which imprecise interpretations are part of the truth conditions of sentences containing numerical expressions, as opposed to being cases of pragmatic slack, where a speaker says something that is literally false but 'close enough' to true (Lasersohn 1999). We will revisit this point briefly in Section 6 below.

[2] Note that (14) and (15) separate specifically in the case where the set I is infinite.

Turning to approximators such as *about*, these can be analyzed per Solt (2014) as referring to coarse-grained degrees, that is, intervals around the precise denotation of the modified numeral, with these intervals to be understood as unitary wholes. The width of the interval is again determined at the interpretation index i. Thus the semantics of approximator-containing examples are the following:

(18) Lisa has about 50 sheep. $max\{n : \text{Lisa has } n \text{ sheep}\} \in [50 - k_i, 50 + k_i]$

(19) Lisa has more than about 50 sheep. $max\{n : \text{Lisa has } n \text{ sheep}\} > [50 - k_i, 50 + k_i]$

Crucially, with these definitions, no relation of 'more informative than' (i.e. 'definitively stronger than') obtains between (16) and (18), or between (17) and (19). The denotation of the approximator-modified numeral corresponds to an interval around the precise point-based denotation of the numeral, but the width of this interval can be wider or narrower according to interpretation i; in the limiting case, $k_i = 0$, and the interval reduces to a single point. Conversely, when the granularity parameter is set to be a coarse-grained one, the bare numeral itself can be used to describe a value that deviates from its precise denotation. Essentially, bare and approximator-modified numerals are on this analysis (potentially) equivalent.[3] Thus there is no numerical value that definitively (i.e. across interpretations i) can be described as *about fifty* but not *fifty* (nor vice versa).

However, I propose that the usage of an overt approximator has the effect of establishing the context to be one in which bare numerals are interpreted precisely, while approximators introduce a non-trivial range around the precise point. This corresponds to the interpretations depicted in (3). Conceptually, the idea is that the choice to use an approximator is a conversational move which establishes that imprecision (if intended) will be overtly signaled. Formally, this may be modeled as an update to the common ground in which the set I of interpretations is restricted to a subset I' including just those interpretations i in which bare and approximator-modified numerals are interpreted in this way. Restricted to such interpretations, (16) asymmetrically entails (18), while (19) asymmetrically entails (17).

Let us return now to the polarity-based restrictions on the distribution of approximators that are the topic of this paper. Recall that the paradigm of interest is the following: in simple numerical constructions, approximators such as *about* are PPIs, whereas when embedded in comparative quantifiers they are NPIs:

(20) a. Lisa has / *doesn't have about 50 sheep.
b. Lisa *has / doesn't have more than about 50 sheep.

In each of these cases, the sentences with the approximator competes with the alternative formed by deleting the approximator. The later alternative has an advantage in terms of simplicity. Furthermore, as discussed above, there is no difference – in the relevant sense – in informativity. Thus the unmodified alternative is the better one overall. This is represented formally below for the sentences in (20a):

[3] I thank Benjamin Spector for suggesting this characterization of the pattern in question.

(21) ϕ = *Lisa has about 50 sheep* ϕ' = *Lisa has 50 sheep*

$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

$\rightsquigarrow$ ϕ' is not weakly assertable ✔

(22) ϕ = **Lisa doesn't have about 50 sheep* ϕ' = *Lisa doesn't have 50 sheep*

$\phi \sim_{INF} \phi'$ $\phi' \succ_{SIMP} \phi$ $\phi' \succ \phi$

$\rightsquigarrow$ ϕ' is not weakly assertable **contradiction**

By the conversational principle in (5), the result is that the utterance of one of the approximator-modified sentences in (21) and (22) gives rise to the implicature that the speaker could not have asserted its unmodified alternative. But as shown above, the effect of this implicature is different in the positive and negative cases. At the contextually available interpretation indices *i*, the positive ϕ in (21) is asymmetrically entailed by its better alternative ϕ', and thus the assertion of the former results in the acceptable implicature that the speaker was not in a position to assert that '(exactly) 50' obtains. But the negative ϕ in (22) asymmetrically entails its alternative ϕ', with the result that the corresponding implicature contradicts the original assertion. This results in blocking, and thus the PPI status of approximators in simple sentences.

The account works in parallel for cases with comparative quantifiers, but since the entailment relation between the approximator-modified numeral and its unmodified alternative is reversed, so too is the observed pattern of polarity sensitivity. As in the simple numerical case in (21)-(22), the 'about' sentence competes with the alternative formed by deleting the approximator, with the latter better overall, because it is simpler and not (definitively) less informative. We thus obtain the implicatures shown below. That in (24) is informative (the speaker isn't in the position to assert that 'no more than exactly 50' obtains), whereas that in (23) is contradictory and results in blocking.

(23) ϕ = *Lisa has more than about 50 sheep*

$\rightsquigarrow$ ϕ' = *Lisa has more than 50 sheep* is not weakly assertable **contradiction**

(24) ϕ = *Lisa doesn't have more than about 50 sheep*

$\rightsquigarrow$ ϕ' = *Lisa doesn't have more than 50 sheep* is not weakly assertable ✔

Thus the present account explains the polarity sensitivity of approximators as arising from contradictory implicatures relative to an unmodified alternative, which is simpler and therefore (in this case) formally better than the original sentence. The account can be extended to other approximators such as *approximately* and *roughly*, if these are taken to have interpretations parallel to *about*, and to other numerical constructions. For example, we correctly predict the ungrammaticality of **Lisa has over about 50 sheep*, since as in the comparative quantifier case it gives rise to a contradictory implicature.

In the next section, we will look in more depth at the nature of the implicatures generated in the grammatical uses of approximators, where we will see one case that patterns differently from *about*.

4. Approximators and implicature

As described in the preceding section, the use of an approximator-modified numerical expression gives rise to an implicature that the corresponding unmodified assertion is not weakly assertable, that is, that it is not the case that the speaker believes it to be true, relevant and supported by the facts. For example:

(25) Lisa has about 50 sheep
$\rightsquigarrow$ the speaker isn't in the position to assert 'Lisa has (exactly) 50 sheep'

This is an example of what Meyer (2013) calls a **weak implicature**, that is, an implicature that the speaker does not know that ϕ. It is distinct from a true **ignorance implicature**, i.e. an implicature that the speaker neither knows that ϕ nor knows that $\neg\phi$. This correctly captures the facts: (25) could be asserted by a speaker who knows Lisa has (say) 47 sheep but chooses to round off to the more salient value 50; however, it is less felicitous if the speaker knows that *exactly 50* obtains. The rounding use is possible for a range of approximators, as evidenced by the felicity of the following, which might be uttered by the principal investigator of the study in question:

(26) About / roughly / around / approximately 70 patients took part in the clinical study.

Other approximating constructions, however, have a stronger ignorance effect, an example being approximating disjunctions, as illustrated in (27a); we would tend to be concerned if the researcher described her study as in (27b):

(27) a. Lisa has 40 or 50 sheep $\rightsquigarrow$ the speaker doesn't know the exact number.
b. ?? 60 or 70 patients took part in the clinical study.

The present framework derives this ignorance implicature in a similar way to other accounts of ignorance effects with disjunctions (e.g. Katzir 2007). Approximating disjunctions of the form in (27a) have the two individual disjuncts as alternatives, as shown in (28). Both of these alternatives are better than the original sentence, being simpler and arguably more informative.[4] As described in Solt (2016), I take the use of such disjunctions to establish that the contextual level of granularity at which numerical expressions are interpreted is equal to the gap between the two values. In this case, this is $gran = 10$, i.e. *40* is interpreted as 40 ± 5 and *50* is interpreted as 50 ± 5. Formally, this again may be modeled via a restriction on the set I of available interpretations. We thus derive the implicatures in (29), which taken together amount to an ignorance effect.

[4] In fact, in order for a 'definitively stronger' relation to obtain between these alternatives, it is necessary to assume some upper bound on the degree to which bare numerals can be interpreted approximately, as otherwise any value that could be described as *forty or fifty* could also be described as either *forty* or *fifty*. I believe this is plausible, though I have to leave a formal derivation to future work. In any case, even without informativity, the individual disjunct alternatives would be better than the original disjunction on account of their greater simplicity.

(28) ϕ = *Lisa has 40 or 50 sheep.* $max\{n : \text{Lisa has } n \text{ sheep}\} = 40_{gran=10} \cup 50_{gran=10}$
ϕ' = *Lisa has 40 sheep.* $max\{n : \text{Lisa has } n \text{ sheep}\} = 40_{gran=10}$
ϕ'' = *Lisa has 50 sheep.* $max\{n : \text{Lisa has } n \text{ sheep}\} = 50_{gran=10}$

$\phi'/\phi'' \succ_{INF} \phi$ $\phi'/\phi'' \succ_{SIMP} \phi$ $\phi'/\phi'' \succ \phi$

(29) a. $\rightsquigarrow$ The speaker doesn't hold the belief that $40_{gran=10}$ obtains
($\equiv$ the speaker doesn't hold the belief that $50_{gran=10}$ doesn't obtain)
b. $\rightsquigarrow$ The speaker doesn't hold the belief that $50_{gran=10}$ obtains
($\equiv$ the speaker doesn't hold the belief that $40_{gran=10}$ doesn't obtain)

Finally, up to this point it has been sufficient to consider alternatives derived via deletion of some part of the original sentence. On the structural approach adopted here, alternatives may also be derived via substitution. This is relevant in particular in the case of approximators in comparatives, where it accounts for certain additional interpretive effects. For example, in some contexts the utterance of (30a) would convey that the speaker considers it possible that Lisa has about 50 sheep. This inference can be accounted for by taking into consideration a fuller set of alternatives such as those in (30b), derived via deletion the approximator and/or substitution of *50* by other values of a comparable level of granularity. In particular, the alternative obtained by replacing *50* by *40* is a better one than the original (30a), being equal in terms of structural simplicity and more informative; see (31). Thus the use of (30a) generates the implicature that this alternative is not weakly assertable.

(30) a. ϕ = *Lisa doesn't have more than about 50 sheep.*
b. $ALT(\phi) = \{\text{Lisa doesn't have more than about } n \text{ sheep} : n = \ldots 40, 50, 60, \ldots\}$
$\cup \{\text{Lisa doesn't have more than } n \text{ sheep} : n = \ldots 40, 50, 60, \ldots\}$

(31) ϕ = *Lisa doesn't have more than about 50 sheep.*
ϕ' = *Lisa doesn't have more than about 40 sheep.*

$\phi' \succ_{INF} \phi$ $\phi' \sim_{SIMP} \phi$ $\phi' \succ \phi$

$\rightsquigarrow$ The speaker doesn't believe that Lisa doesn't have more than about 40 sheep, i.e. considers it possible that she has about 50.

This is a variety of scalar implicature. Below we will see that the possibility of generating this implicature sets the present account apart from a possible alternative.

5. Alternate analytical approaches

The present account of the polarity sensitivity of approximators has been framed in a pragmatic theory of implicatures (Krifka 1995, Katzir 2007), the central component of which is a conversational principle or rule of assertion that governs the choice between alternative expressions a speaker could use. The leading competitor to such an approach is the grammatical theory championed by authors including Chierchia (2004, 2006, 2013), Fox (2007), Spector (2014) and others, under which implicatures (and polarity-based distributional restrictions) derive from the operation of a covert exhaustification operator. In this section, I briefly consider how the empirical data discussed here might be handled on such an approach.

In one implementation of the grammatical theory, Chierchia makes use of the operator *O*, a silent counterpart of overt *only*, whose effect is to negate all non-entailed alternatives of a sentence containing an alternative-introducing element.

(32) $[\![O]\!] = \lambda p \lambda w.[p_w \wedge \forall q \in ALT(p)[q_w \rightarrow p \subseteq q]]$

The operation of *O* accounts for scalar implicatures as well as the polarity sensitivity of items such as *any*. However, a straightforward extension of its application to approximators yields incorrect results. Suppose, for example, that the alternatives to *about 50* are taken to be the individual values in the range denoted by *about 50*, as in (33)-(34). Then application of *O* in the positive case yields a contradiction, as these alternatives fully cover the semantic territory of the approximator-modified numeral; the sentence is thus predicted to be ungrammatical. The negative sentence, on the other hand, is predicted to be grammatical, as all of its alternatives are entailed. This is of course the opposite of the pattern that is actually observed.

(33) a. p=Lisa has about 50 sheep.
$ALT(p) = \{$Lisa has n sheep : $n \in 50 \pm k_i\}$
b. *O*(Lisa has 50 sheep) $\rightsquigarrow$ contradiction!

(34) a. p=Lisa doesn't have about 50 sheep.
$ALT(p) = \{$Lisa doesn't have n sheep : $n \in 50 \pm k_i\}$
b. *O*(Lisa doesn't have 50 sheep) $\rightsquigarrow$ vacuous

If instead we assume that the only alternative is that based on the numeral *50* on its precise interpretation (similarly to what was done in the present approach), the positive sentence is predicted to be grammatical, but we incorrectly generate the implicature that 'exactly 50' does not obtain. As discussed in the preceding section, the actual implicature is a weaker one, namely that 'exactly 50' is not assertable by the speaker. And once again, the negative sentence is predicted to be fully acceptable, since this single alternative is entailed.

More promising results are obtained by incorporating some more recent proposals, in particular the notion of obligatory exhaustification introduced by Spector (2014) as well as the covert doxastic operator *K* of Meyer (2013), where *Kp* can be interpreted as 'the speaker believes that *p*'. Assuming an exhaustification operator *exh* with broadly speaking the semantics of the *O* operator above, and taking sentences with approximators to have a single alternative formed by deleting the approximator, we correctly derive the following results for simple sentences: the positive (35) is grammatical, and has the implicature that the speaker does not know that 'exactly 50' obtains; the negative (36) is blocked, because *exh* would be vacuous.

(35) $exh(K(\text{Lisa has about 50 sheep}))$
$ALT(p) = \{K(\text{Lisa has } 50_{EXACT} \text{ sheep})\}$
$\rightsquigarrow K(\text{Lisa has about 50 sheep}) \wedge \neg K(\text{Lisa has } 50_{EXACT} \text{ sheep})$ **correct**

(36) $exh(K(\text{Lisa doesn't have about 50 sheep}))$
$ALT(p) = \{K(\text{Lisa doesn't have } 50_{EXACT} \text{ sheep})\}$
$\rightsquigarrow$ Single alternative is entailed; *exh* is vacuous. **correct**

However, other results are less satisfactory. In particular, consider again a comparative example such as *Lisa doesn't have more than about 50 sheep*, which as discussed above tends to implicate that the speaker considers it possible that Lisa has about 50 sheep. On the approach developed in the present paper, this inference is accounted for as a scalar implicature, by taking into consideration not just the unmodified alternative but also those such as *Lisa doesn't have more than about 40 sheep.* This could be accommodated in a parallel way in the grammatical approach. But when such alternatives are also included in the case of the positive comparative sentence, *exh* is no longer vacuous, and the sentence is predicted to be acceptable. For example, *Lisa has more than about 50 sheep* would have the non-entailed alternative *Lisa has more than about 60 sheep*, to which the exhaustification operator could apply non-vacuously. To salvage this approach, it seems we would need to specify that *exh* is obligatory specifically with respect to the unmodified alternative. This is equivalent to building a preference for simplicity into the system, just as done here.

This brief discussion has barely scratched the surface in terms of the analytical possibilities available under a grammatical theory of implicature and polarity sensitivity. I have no doubt that the insights that form the basis of the account developed in the present paper could also be implemented within such an approach. However, I hypothesize that doing so will require incorporating some of the same formal innovations that were adopted in present account.

6. Conclusions and future direction

In this paper, I have proposed a pragmatic account of the polarity sensitivity of approximators such as *about*, according to which their ungrammatical occurrences are analyzed as being due to blocking by the unmodified form, or equivalently, an implicature that contradicts the asserted content of the utterance. I have framed the analysis in a formal system whose central components are a rule of assertion that calls for the comparison of alternatives in terms of both informativity and simplicity, and a novel definition of relative informativity as it pertains to sentences containing vague predicates.

There are a number of issues that due to space limitations I have not been able to pursue here. In particular, I have not considered approximators in NPI-licensing contexts other than sentential negation, nor have I been able to go into much depth regarding potential difference between approximating constructions. One sort of example that merits further investigation involves approximating disjunctions. The present analysis would seem to predict incorrectly that *Lisa doesn't have more than 40 or 50 sheep* should be ungrammatical, as its alternative *Lisa doesn't have more than 50 sheep* is semantically equivalent but simpler. Intuitively, what makes the disjunctive example acceptable is that its implicatures are different than those of its simpler alternative, in that the former but not the latter conveys that the speaker thinks it possible that Lisa has 40 or 50 sheep. This suggests that the system developed here may need to be refined to take into consideration not only the semantic interpretation of the alternatives to a given sentence, but also their pragmatic inferences. Indeed, such a refinement may be necessary even for the data considered in the present paper, if we take a pragmatic rather than semantic approach to the imprecise interpretations of bare numerals. I leave the development of such a refinement to future work.

Also of interest is how the present approach might be extended to cases beyond approximators. Many other members of the class of attenuating polarity items (e.g. *much*, *fairly*) are also characterized by their vagueness, suggesting that they too might be amenable to an analysis in terms of (lack of) informativity relative to the corresponding unmodified forms. At the same time, not all vague modifiers are polarity sensitive, an example being *very*, discussed briefly in Section 2. Thus an important challenge will be to characterize what specifically sets apart the polarity sensitive class.

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