Precision, Vagueness, Scales and the Back-down Phenomenon*

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Vague Quantities and Vague Quantifiers (VQ2)

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*Special thanks to Brendan Gillon.

Outline



2 Heaps and Heaps of Paradox!

- 3 Is MOST vague?
- 4 Sorites with MOST?
- 5 Consequences for Scales

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Precision, Vagueness, Scales and the Back-down Phenomenon Loose Talk and Vagueness

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Loose Talk vs. Imprecise Meanings

LOOSE TALK: Precise meanings that are used imprecisely either to facilitate pragmatic implicatures (e.g., exaggeration) or avoid tedious and indepth verification procedures (e.g., estimation).

IMPRECISE MEANINGS: Imprecision that is part of the literal meaning of a sentence/term.

Precision, Vagueness, Scales and the Back-down Phenomenon Loose Talk and Vagueness

Loose Talk vs. Imprecise Meanings

LITERAL MEANING = Truth Conditions

VERIFICATION PROCEDURE =

Methods to check if Truth Conditions are met

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Exaggeration and Loose Talk

- (1) a. Everyone is wearing bows in her hair. (Uttered recently by my daughter)
 - b. I got a thousand emails today. (Uttered by me to my colleague Charles Reiss)
 - c. There are millions of things we could do to fix the window. (Uttered by me to my wife)

The Jerk's role in Diagnosing Loose Talk

The interesting thing about loose talk is that people back-down when challenged by a stickler (i.e., jerk).



"All we want are the facts, ma'am"

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Precision, Vagueness, Scales and the Back-down Phenomenon —Heaps and Heaps of Paradox!

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Precision, Vagueness, Scales and the Back-down Phenomenon —Heaps and Heaps of Paradox!

(Perceptual) Indistinguishability

Indistinguishable Height:

 If I cannot distinguish the height between two individuals by looking at them, then I tend to treat them as if they are equally as tall. Let's symbolize this indistinguishability relation as ~T.

(Perceptual) Indistinguishability

Important observation (see, van Rooij, 2010; Cobreros, Egré, Ripley & van Rooij, 2010):

■ The relation \sim_T is not an equivalence relation. In particular it is not transitive. If $a \sim_T b$ and $b \sim_T c$, it does not follow that $a \sim_T c$. For example, suppose that I cannot perceptually distinguish the difference between two heights when they are only 1mm apart but I can when they are 2mm apart. Suppose *a* is 1802mm tall, *b* is 1801mm tall and *c* is 1800mm tall. Thus, $a \sim_T b$ and $b \sim_T c$ but I can clearly distinguish *a* from *c*.

(Perceptual) Indistinguishability

TWO TYPES OF TRUTH:

- **Literal Truth**: A predicate P is (literally) true of an individual a iff $a \in [\![P]\!]$.
- **Tolerant Truth**: A predicate P is tolerantly true of an individual a iff there is some individual b that is perceptually indistinguishable from a with respect to the property P and $b \in \llbracket P \rrbracket$.

Sorites

- Premise P1: For any individuals x and y, if x is classified as tall and y is perceptually indistinguishable from x in terms of height, then y is classified as tall. Two possible versions: one under a "tolerant" implication, another under a literal implication.
 - Literal P1: ∀x, y((Tx & x ~_T y) → Ty)
 Tolerant P1: ∀x, y((Tx & x ~_T y) →_t Ty), where for any two propositions, p and q, p →_t q is true iff either p is not literally true or q is tolerantly true.*

*Note that \rightarrow_t is not transitive. If p is literally true and we know that $p \rightarrow_t q$ and $q \rightarrow_t r$, we can conclude that q is tolerantly true but we cannot conclude either that r is literally true or tolerantly true. To conclude r, we would need q to be literally true.

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Sorites

- **Premise P2** (Given by Assumption): There is a line of people from $a_1 a_2 a_3 \dots a_n$ such that
 - **1** a_1 is (literally) tall.
 - **2** a_n is (literally) not tall.
 - **3** For any two adjacent members of the line, a_i and $a_{(i+1)}$, the difference in height between the two is not perceptually distinguishable (i.e., $a_i \sim_T a_{(i+1)}$).

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Sorites

- From Literal P1 and P2, one arrives at the contradiction $(Ta_n \& \neg Ta_n)$.
- From **Tolerant P1** and **P2**, there is no contradiction. One cannot conclude either that Ta_n is literally true or tolerantly true. This is due to the intransitivity of \rightarrow_t .

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Precision, Vagueness, Scales and the Back-down Phenomenon LIS MOST vague?

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ANS and MOST

Pietroski, Lidz, Hunter & Halberda (2009) demonstrate that people engage their Approximate Number System (hereon ANS) when evaluating sentences with the quantifier *most*.

(2) Most of the dots are blue.

They conclude that either the semantics of *most* is represented using ANS or the semantics is represented using cardinality but the verification procedures use ANS as an estimation of cardinality.

- (3) Two Potential Truth Conditions for (2)
 - a. $|D \cap B| > |D B|$
 - b. $approx(D \cap B) \triangleright approx(D B)^*$

*Where *appox* is a function from sets to an Approximate Number Representation and >> is the greater-than relation with respect to ANRs.

Precision, Vagueness, Scales and the Back-down Phenomenon $$L_{\rm Is\ MOST\ vague?}$$





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(4) Most of the dots are blue.

Precision, Vagueness, Scales and the Back-down Phenomenon $$L_{\rm Is\ MOST\ vague?}$$





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(5) Most of the dots are blue.

ANS and MOST

If ANS is part of the semantic interpretation, then this would make *most* vague since ANS comparisons are vague (see Fults, forthcoming). But, challenges from a stickler (i.e., jerk) demonstrate that *most* is not vague and thus that the proper semantic representation does not involve ANS.

Precision, Vagueness, Scales and the Back-down Phenomenon

Most, ANS and Fake Confidence



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(6) Most of the dots are blue.

Precision, Vagueness, Scales and the Back-down Phenomenon

Most, ANS and Fake Confidence



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(7) Most of the dots are blue.

Precision, Vagueness, Scales and the Back-down Phenomenon <u>Sorites with MOST</u>?

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Precision, Vagueness, Scales and the Back-down Phenomenon Sorites with MOST?

Most, ANS and Sorites

■ Perceptually if two large sets are close in number, our ANS cannot distinguish them. Let's use the symbol ~_{ans} to represent this indistinguishability relation. This leads to the potential set-up for a sorites paradox for *most*.

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Most, ANS and Sorites

- Let D_X be the set of 10,000 dots in a given picture that contains the set of blue dots X. Let D_Y indicate the set of 10,000 dots in a given picture that contains the set of blue dots Y. Etc. It is important that for any set of blue dots Z, the cardinality of D_Z is always 10,000.
 - 1 Literal P1: $\forall X, Y((MOST(D_X, X) \& X \sim_{ans} Y) \rightarrow MOST(D_Y, Y)).$
 - **2** Tolerant P1: $\forall X, Y((MOST(D_X, X) \& X \sim_{ans} Y) \rightarrow_t MOST(D_Y, Y)).$
 - **3** P2 Assumption: Consider the following line of dot pictures $a_1 a_2 ... a_n$, each picture containing 10,000 dots. In a_1 , suppose that it is easy to see that most of the dots are blue. In a_n , suppose it is easy to see that less than half of the dots are blue. For any two adjacent pictures, a_i and $a_{(i+1)}$, the set of blue dots in a_i , call them X is indistinguishable in number (via ANS) from the set of blue dots in $a_{(i+1)}$, call them Y. Thus, $X \sim_{ans} Y$.

Estimations and Sorites

Does it seem like a paradox? I think it does, at least in the same way it did for *tall*. But it is easy to determine the way out. Since the semantic representation is not vague, we can use backdown to show where the breakdown could happen for Literal P1. We can also clearly distinguish Literal P1 from Tolerant P1.

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Precision, Vagueness, Scales and the Back-down Phenomenon —Consequences for Scales

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Consequences

Some interesting consequences...

Scalar Vagueness (a.k.a., Granularity and Persification): Should the type of vagueness that arises from granularity of the scale be built into the semantic representation of scales or should it merely be part of the verification procedures? Back-down suggests that it is merely a side-effect of the verification procedures. (c.f. Pinkal, 1995; Bale, 2006; van Rooij, forthcoming; Sauerland & Stateva, forthcoming)

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- (8) a. John is taller than Bill.
 - b. John is as tall as Bill

Vagueness of the Positive Form



Some interesting consequences...

- Scalar Vagueness (a.k.a., Granularity and Persification)
- Vagueness of the Positive Form: Should the type of vagueness that arises in the positive form be built into the semantic representation of scales or should it merely be part of the verification procedure? Back-down suggests that it should be part of the literal meaning (see Fults, forthcoming).

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(9) John is tall compared to Bill.

Precision, Vagueness, Scales and the Back-down Phenomenon —Consequences for Scales



Some interesting consequences...

Scalar Vagueness (a.k.a., Granularity and Persification)

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Vagueness of the Positive Form